SOME PROPERTIES OF THE UNDEREXPANDED SONIC JET ON RETARDATION

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Sharp variation of pressure was observed downstream of the diffusor on investigating the retardation of an underexpanded sonic jet by means of the diffusor. This phenomenon is explained by examining the Schlieren photographs. A relationship was obtained for determining the mass rate of flow through the central shock.

The underexpanded jet has parameters which vary over sections, especially after the central compressive shock where a subsonic core is developed which is surrounded on the periphery by a supersonic flow. In the literature there is a lack of information about stagnation of subsonic or supersonic flow by diffusors with such nonuniformity of parameters. A report is given below about results of application of a subsonic diffusor for retarding an underexpanded sonic jet.

The experiments were carried out on an experimental rig which has a chamber with transparent side walls into which are built a coaxial nozzle and diffusor. At $M_a = 1$ the pressure upstream of the nozzle P_R is regulated according to the experimental program by means of a reducing valve. The relative gap x/d_a between the outlet edge of the nozzle and the inlet to the diffusor can be varied within the range of 0 to 9 by means of a screw arrangement. The ratio between the diameter of the nozzle and the inlet to the diffusor was varied by changing the nozzles within a range from 0.47 to 0.92. The measurement of the temperature was carried out by a thermometer at three points: upstream of the nozzle, downstream of the diffusor, and in the chamber. The flow rate of the air fed into the nozzle G_c and hence not entering into the diffusor G_k, is measured by chamber diaphragms. The stagnation pressure upstream of the nozzle P_R, and downstream

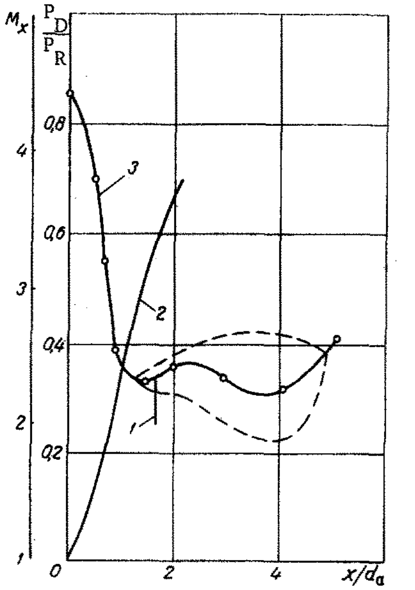

Fig. 1. Coefficient of the recovery of pressure in the diffusor in relation to the relative distance from the outlet edge of the nozzle to the diffusor at $M_a = 1$; $n = 3.7$; $\varkappa = 1.4$: 1) position of the central shock; 2) curve of the variation of the Mach number along the axis of the jet (M); 3) pressure recovery coefficient.

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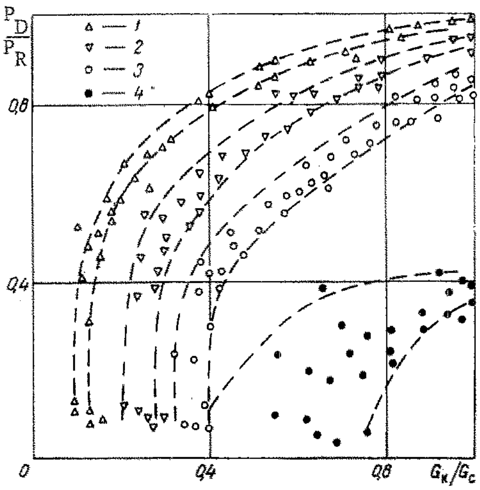

Fig. 2. Coefficient of the pressure recovery in the diffusor in relation to relative flow rate of the lost gas and the relative distance from the outlet edge of the nozzle to the diffusor. 1) $x/d_a = 0.192$; 2) 0.576; 3) 0.963; 4) 1.44.

of the diffusor is found by a standard pressure gage. All measurements were carried out at steady state of the flow. The error of the measurements made was 10-15%.

The visualization of the flow between the nozzle and the diffusor was accomplished by the dark schlieren apparatus IAB-451, which made it possible to investigate the structure of the flow, and record it on a photographic plate.

The results of the experimental work are shown in Fig. 1 in the form of the relationship $P_D/P_R = f(x/d_a)$ for the range of values of P_D, corresponding to those settings of the valve established downstream of the diffusor, in which its maximum closing did not cause further increase in the value of P_D. The zone of maximum recovery of pressure is to the left of the line which characterized the position of the central shock of compression of the first "barrel." From the analysis of the data of P_D/P_R for different values of d_a/d_D it is seen that with the reduction of d_a/d_D the coefficient of pressure recovery will drop sharply.

Results of the measurements of $P_D/P_R = f(x/d_a)$ at n = 3.7 to 10 were compared with the curve of distribution of the M_x on the axis of the jet [1]. From the graph in Fig. 1 it is seen that up to $M_x \leq 1.5$ the P_D/P_R is approximately unity. Hence the relationship x/d_a, which corresponds to $M_x \leq 1.5$, is considered as optimal.

For practical purposes it is important to find the optimum between the weight loss of the gas G_k and the pressure loss ΔP, induced by the nonisentropy of the flow at the inlet to the expanding section of the diffusor.

At the given relationship d_a/d_D the magnitude of the weight losses of the gas will be dependent also on the position of the boundary of the underexpanded jet.

By locating the optimal conditions of stagnation of the sonic jet the instability of the flow which is associated with the peculiarities of the structure of such a jet was detected. In Fig. 1 the zone of instability of P_D is shown by a broken line. It has a significant length and it is positioned in the interval between the central shock of the first "barrel" and the end of the second one. A qualitative picture of such instability is shown in Fig. 3.

The failure of the stable operation of the diffusor was also observed indirectly by measuring P_D/P_R in relation to the relative flow rate of the lost gas G_k/G_c. In Fig. 2 this zone is characterized by wide scatter of the points, by sharp drop of the coefficient of the pressure recovery P_D/P_R, and the increase of the flow of lost gas.

This phenomenon can be explained, apparently, by the fact that at $x \geq x_0$ the flow in the inlet into the diffusor is characterized by marked nonuniformity of the parameters. Directly upstream of the central shock in the zone in the vicinity of the axis of the jet the flow has high velocity and low density; on transition through the central shock this zone of the flow is converted into subsonic flow, the diameter of which is determined by the diameter of the central shock. The subsonic zone is encompassed by the periphery of the supersonic flow. In this zone the pressure of the retarded flow downstream of the diffusor can propagate upwards along the flow of the jet which is seen in Fig. 3b. Opposing flow is formed and, at the instant of the sharply decreased P_D, the diffusor operates as a nozzle.

By using the correspondence with one-dimensional gas dynamics for the flow along the axis of the jet upstream of the central shock, and downstream of it, and also the determined value of q_1

$$\lim q_1 = \lim_{M \to \infty} f_1(M, \varkappa) = \left(\frac{2\varkappa}{\varkappa+1}\right)^{\frac{1}{2}\frac{\varkappa+1}{\varkappa-1}} \left(\frac{\varkappa-1}{2\varkappa}\right)^{\frac{1}{2}},$$

the relative specific flow rate of the gas upstream of the central shock can be represented in the form

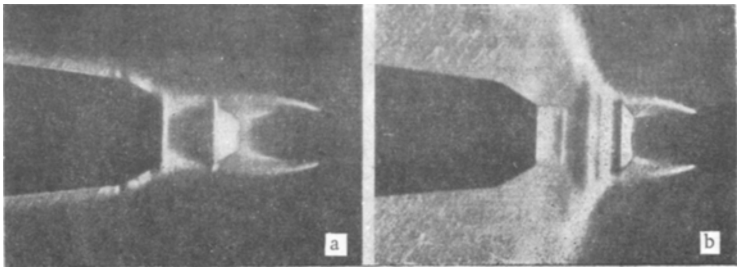

Fig. 3. Schlieren photographs of the air flow between the nozzle and the diffusor (inlet of the diffusor is positioned downstream of the central shock): a) instant of the gradual increase of the pressure downstream of the diffusor; b) instant of the sudden pressure drop downstream of the diffusor.

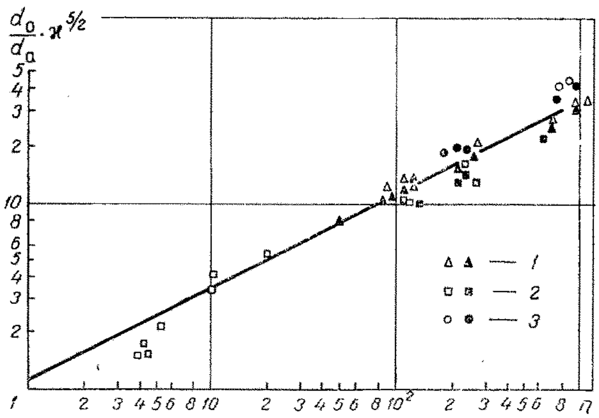

Fig. 4. Relative diameter of the central shock jump in relation to the ratio of pressures $n = P_a/P_\infty$ at $M_a = 1$: 1) $\varkappa = 1.3$; 2) 1.4; 3) 1.67; unfilled points, $T_0 = 289$°K. Dark points, $T_0 = 700$°K.

$$q = \left(\frac{2\varkappa}{\varkappa+1}\right)^{\frac{1}{2}\frac{\varkappa+1}{\varkappa-1}} \left(\frac{\varkappa-1}{2\varkappa}\right)^{1/2} \frac{P_{01}}{P_0}. \tag{1}$$

Assuming that the flow along the axis of the jet up to the central shock is isentropic, and on using the determining relationship P_{01}/P_1

$$\lim \frac{P_{01}}{P_1} = \lim_{M\to\infty} f_2(M, \varkappa) = \left[\frac{(\varkappa+1)^2}{4\varkappa}\right]^{\frac{\varkappa}{\varkappa-1}},$$

and also the established experimental relationship $P_1 \approx P_\infty$ [4], the formula (1) can be written in the form

$$q = \left(\frac{\varkappa+1}{2}\right)^{\frac{3\varkappa-1}{2(\varkappa-1)}} \left(\frac{\varkappa-1}{2}\right)^{1/2} \frac{1}{\varkappa\left(1+\frac{\varkappa-1}{2}M_a^2\right)^{\varkappa/\varkappa-1}} \frac{1}{n} \tag{2}$$

or for $M_a = 1$ (sonic nozzle)

$$q = \left(\frac{\varkappa^2-1}{4\varkappa^2}\right)^{1/2} \frac{1}{n}. \tag{2a}$$

Expressions (2) and (2a) are correct when the Mach number upstream of the central shock $M \gg 1$, which corresponds to a value of $n > 5$ to 7.

Using the empirical relationship for relative diameter of the central shock of the sonic jet

$$\frac{d_0}{d_a} = \frac{1.1}{\varkappa^{5/2}} n^{0.5}, \tag{3}$$

which is obtained from the work [5]* (see Fig. 4), and the expression (2a), can show that an insignificant part of the overall mass flow of the gas which is discharged from the nozzle passes through the central shock.

We will assume that the parameters of the gas at any given point of the cross section of the jet are equal to the parameters on the axis of the jet which correspond to isentropic flow in some ideal Laval nozzle. In this case the expression (2a) represents a relationship between an area of critical section (in our case the areas of a sonic nozzle at the outlet edge) and the plane of the section of the assumed Laval nozzle at the location of the central shock

$$\frac{1}{q} = \frac{A_{pr}}{A_*} = \frac{d_{pr}^2}{d_a^2} = \left(\frac{4\varkappa^2}{\varkappa^2-1}\right)^{1/2} n. \tag{4}$$

*The influence of T_0 on the magnitude of the diameter of the central shock $M_a = 1$ is insignificant in the interval $T_0 = 289$ to 700°K and it can be ignored.

On squaring the expression (3) and on substituting the result into (4) we will obtain a relationship for that part of the mass flow of the gas which passes through the central shock

$$\frac{\Delta G_0}{G} = \frac{d_0^2}{d_a^2} \Big/ \frac{d_{pr}^2}{d_a^2} = 1.2 \left(\frac{\varkappa^2 - 1}{4\varkappa^{12}} \right)^{1/2} = \psi(\varkappa). \tag{5}$$

Hence we assume that the flow directly upstream of the central shock has parameters which are constant over the section and equal to the parameters on the axis.

The formula (5) shows that the mass flow of the gas through the central shock depends basically on the magnitude of ψ, which is given as the function of the ratio of specific heats $\varkappa = c_p/c_y$.

Calculation shows that the magnitude of $\psi(\varkappa) \ll 1$ and for $\varkappa = 1.3$ to 1.67 is correspondingly within the range 0.10-0.04.

Consistent with experimental data obtained by processing of Schlieren photographs of the jet [5], the ratio between the section occupied by the central shock and the maximum section of the jet for $M_a = 1$ can have significant values, particularly at high values of n and at low $\varkappa$:

$$\frac{d_0^2}{D_{max}^2} = 0.3 \frac{1}{\varkappa^3} n^{0.14}. \tag{6}$$

The relationship (6) satisfactorily agrees with experimental values for variation of n from 4 to 10^3 and $\varkappa = 1.3$ to 1.67. Hence in the central zone of the jet determined by the diameter of the central shock the mass flow of the gas is insignificant; the basic mass of the gas flows in the circular zone which surrounds the central shock.

In all probability, the unstable regime which occurs during retarding of the jet can be eliminated by means of applying an annular diffusor.

NOTATION

x	is the coordinate along the axis from nozzle outlet edge;
n	is the degree of underexpansion;
M	is the Mach number;
P	is the gas pressure;
ρ	is the gas density;
T	is the absolute gas temperature;
$\varkappa = c_p/c_v$	is the ratio of specific heats;
d_0	is the diameter of the central shock;
G	is the mass flow rate;
d_a	is the diameter of the nozzle outlet section;
A	is the cross section area;
q	is the relative specific flow rate;
D_{max}	is the maximum diameter of the first "barrel" of the jet;
d_D	is the diameter of diffusor inlet section;
x_0	is the distance from the nozzle edge to central shock.

Subscripts

1	gas parameters downstream of the primary shock;
a	in the nozzle outlet section;
∞	in the ambient medium;
0	isentropic retardation;
01	retardation downstream of the primary shock;
D	downstream of the diffusor;
R	in the receiver;
c	nozzle;
*	critical section.

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